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GENERALIZED HEAT TRANSFER AND FRICTION CORRELATIONS FOR TUBES WITH REPEATED-RIB ROUGHNESS

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NOMENCLATURE

D, pipe inside diameter (to base of rib);
D_{eq}, pipe equivalent diameter $(D - e)$;
D'_{eq}, defined by Hall [8];
e, rib height;
e^+, roughness Reynolds number, $e^+ \equiv eu^*/\nu = (e/D)Re\sqrt{(f/2)}$;
f, rough tube friction factor, $(\Delta P/L)\,D/2\rho u_m^2$;
$\tilde{f}$, rough tube % friction factor based on D_{eq};
$\bar{g}$, $[(f/2St - 1)/\sqrt{(f/2)} + u_e^+]Pr^{-0.57}$ (repeated-ribs);
p, distance between repeated-ribs;
Pr, Prandtl number;
Re, Reynolds number, Du_m/ν;
$\tilde{R}e$, Reynolds number based on D_{eq};
St, rough tube Stanton number;
u, local fluid velocity;
u_m, average fluid velocity;
u^*, friction velocity, $\sqrt{(\tau_0/\rho)}$;
u_e^+, $\sqrt{(2/f)} + 2{\cdot}5 \ln(2e/D) + 3{\cdot}75$;
$\tilde{u}_e^+$, $\sqrt{(2/f)} + 2{\cdot}5 \ln[2e/(D - e)] + 3{\cdot}75$;
y, coordinate distance normal to surface;
τ_0, apparent wall shear stress, $(D/4)(dP/dx)$.

INTRODUCTION

IN A PREVIOUS publication [1] correlations are presented for the friction factor and Stanton number of "repeated-rib" roughness in turbulent pipe flow. Figure 1 shows a sketch of this geometry and defines the roughness parameters as p/e and e/D; two dimensionless parameters should be sufficient,

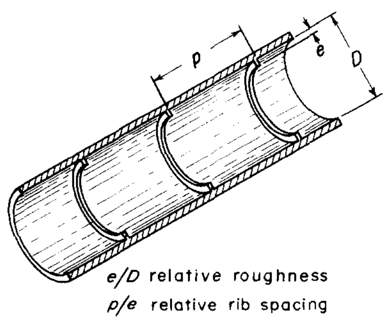

FIG. 1. Sketch of the roughness geometry.

providing the rib thickness is small relative to the rib spacing and not larger than the rib height. Repeated-rib roughnesses are described as "geometrically similar" if p/e = constant and the rib shape is not varied.

The friction data for geometrically similar repeated-rib roughness are correlated in the form u_e^+ vs. e^+, where

$$u_e^+ \equiv \sqrt{(2/f)} + 2{\cdot}5 \ln (2e/D) + 3{\cdot}75. \tag{1}$$

This correlation results from integration of an assumed logarithmic velocity distribution, $u/u^* = 2{\cdot}5 \ln (y/e) + u_e^+$. This is the same correlation method used by Nikuradse [2] for sand grain roughness, although different values of u_e^+ are found for the repeated-rib and sand grain roughness. The velocity distribution law implies that u/u^* is independent of the pipe diameter, so the friction correlation should hold for a range of e/D below some maximum value.

The heat transfer correlation is based on application of a heat-momentum transfer analogy to rough surface flow. This correlating method was developed by Dipprey and Sabersky [3] and successfully applied to the correlation of heat transfer in tubes having sand grain type roughness. The repeated-rib heat transfer data of [1] are correlated within ± 9 per cent by $\bar{g}(e^+)$ vs. e^+, where

$$\bar{g}(e^+) \equiv \left[\frac{f/2St - 1}{\sqrt{(f/2)}} + u_e^+\right] Pr^{-0{\cdot}57}. \tag{2}$$

The friction and heat transfer correlations which were found for the repeated-rib roughness are based on data for $0{\cdot}01 < e/D < 0{\cdot}04$ and $10 < p/e < 40$, with a Prandtl number range of 0·71–37·6.

The present study extends the range of validity of the friction and heat transfer correlations. The data of other investigators, who have worked with both larger and smaller values of e/D, is correlated using equations (1) and (2) and compared to the correlations presented in [1].

COMPARISON WITH OTHER INVESTIGATORS

Gargaud and Paumard [4], Nunner [5] and Molloy [6] measured the heat transfer and friction characteristics of turbulent air flow in pipes having repeated-rib roughness. Gargaud and Paumard's work was done with considerably higher Reynolds number flow and smaller e/D than were studied in [1]. Nunner used larger values of e/D, and Molloy's data were taken at smaller values of e^+, with $e/D = 0{\cdot}014$. Both Gargaud and Paumard, and Nunner using "square-edged" ribs, while Molloy's ribs were of circular cross-section.

Figure 2 shows smoothed friction data of the above investigators presented in the form u_e^+ vs. e^+, where the friction factor in equation (1) was obtained from pressure

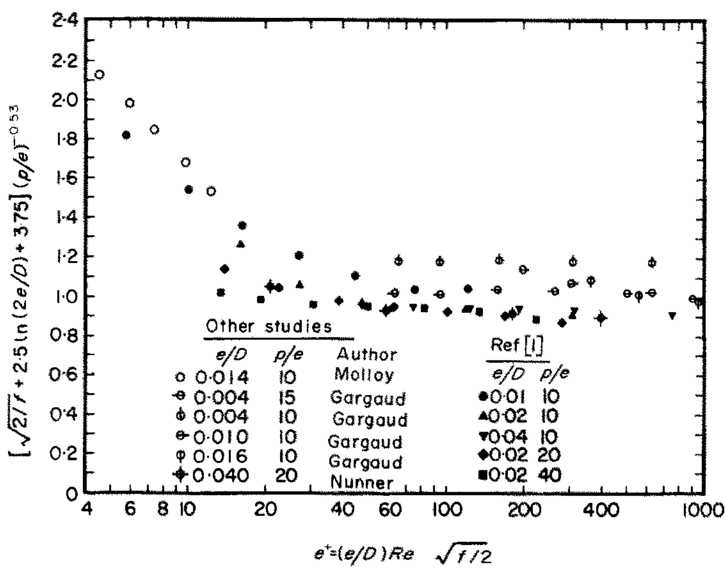

FIG. 2. Correlated friction data of Nunner [5], Molloy [6], Gargaud and Paumard [4] and Webb *et al.* [1].

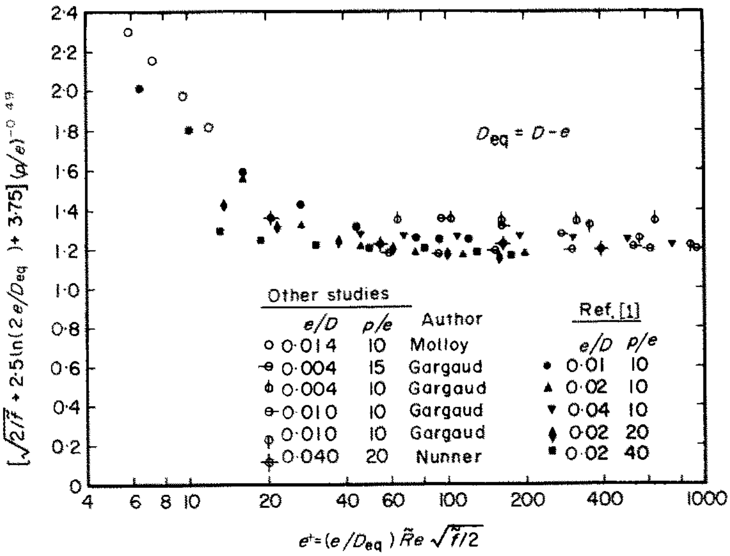

FIG. 3. Correlated friction data of Nunner [5], Molloy [6], Gargaud and Paumard [4] and Webb *et al.* [1] using $D_{eq} = D - e$.

drop measurements. For comparison the solid symbols of Fig. 2 show the friction data of [1]. The results of Nunner and Molloy agree well with the results of [1], but Gargaud and Paumard's results for the smaller e/D are about 15 per cent higher. As discussed in [1], the base surface is seldom taken as the origin for the velocity distribution in rough surface measurements. Figure 3 shows the data of Fig. 2 recorrelated taking the origin at $y = 0{\cdot}5e$; thus $D_{eq} = (D - e)$, and Re and f are based on this D_{eq}. A better correlation of these data is obtained with the scatter reduced to ± 6 per cent. The u_e^+ for Molloy's circular ribs is about 8 per cent larger than that for square edged ribs on both Figs. 2 and 3. This means that the friction factor for circular ribs is about 7 per cent smaller than for square edged ribs.

Figure 4 shows the heat transfer results of Gargaud and Paumard correlated by equation (2), using values of u_e^+

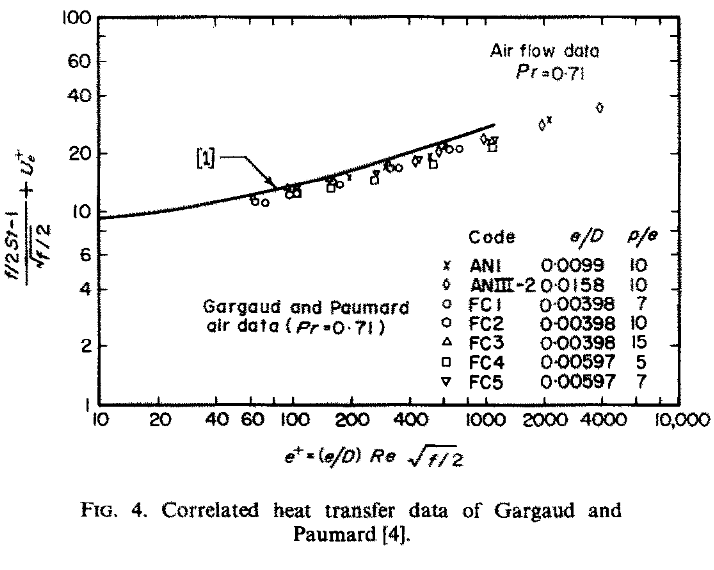

FIG. 4. Correlated heat transfer data of Gargaud and Paumard [4].

obtained from Fig. 2. Figure 4 includes data for $p/e = 5$, which are on the low side of the correlation. The correlated data are from 0 to 9 per cent below the solid line, which represents the $Pr = 0{\cdot}71$ data of [1]. Figure 5 shows the same heat transfer correlation applied to the data of Nunner, and Molloy. Their results agree well with those of [1] (represented by the solid line). Figure 5 also includes Nunner's data for ribs of semi-circular cross-section. For $e/D = 0{\cdot}04$ and $p/e = 20{\cdot}4$, the friction factor for the semi-circular rib is 26 per cent smaller, and u_e^+ is 24 per cent larger than for the square-edged ribs. However, using the appropriate values of u_e^+, the heat transfer correlation is not significantly different for the two rib shapes.

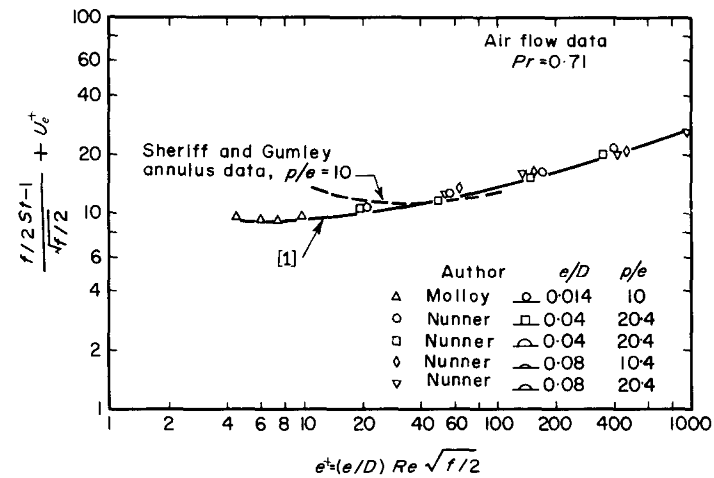

FIG. 5. Correlated heat transfer data of Molloy [6], Nunner [5] and Sheriff and Gumley [7].

The dashed line of Fig. 5 shows the correlation obtained by Sheriff and Gumley [7], who worked with repeated-rib roughness in an annular flow geometry, in which the inner annulus surface was rough and the outer surface was smooth. This correlation is shown because considerable data exist for the annular flow geometry. The Reynolds number and friction factor are based on an equivalent diameter (D'_{eq}) proposed by Hall [8]. Hall theorizes that annular data based on the derived e/D'_{eq} should agree with the results obtained in pipe flow for geometrically similar roughness, where $e/D'_{eq} = e/D$. Sheriff and Gumley's data cover the range $0{\cdot}001 < e/D_{eq} < 0{\cdot}02$, with $p/e = 10$. It appears that this "transformed" annular flow data, based on e/D'_{eq}, agree with the pipe flow correlation if $e^+ > 35$.

The heat transfer data of Figs. 4 and 5 were also correlated using f and Re based on $D_{eq} \cong (D - e)$, and the quality of the correlation differed negligibly from the results of Figs. 4 and 5.

CONCLUSIONS

1. The validity of generalized heat transfer and friction correlations for repeated-rib roughness is further substantiated by their success in correlating the data of several other investigators, which included data in the range $0{\cdot}004 < e/d < 0{\cdot}08$.
2. For p/e in the range of 10–20, a slightly better friction correlation is obtained if the friction factor and Reynolds number are defined in terms of the equivalent diameter, $D - e$.
3. The validity of transforming repeated-rib annulus data, using an equivalent diameter, to an equivalent pipe flow geometry, appears justified for $e^+ > 35$.

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TRANSPIRATION COOLING OF TANGENTIAL NEWTONIAN FLOW IN ANNULI: ANALYTICAL SOLUTIONS FOR TEMPERATURE DISTRIBUTIONS

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NOMENCLATURE

C_p, specific heat of fluid;
J_m, Bessel function of the first kind (mth order);
k, ratio of radius of inner cylinder to that of outer cylinder;
$\bar{k}$, thermal conductivity of fluid;
r, radial co-ordinate;
R_0, radius of outer cylinder;
t, time;
U, variable defined by equation (14);
v, local velocity;
Y_m, Bessel function of the second kind (mth order);
$Z_m(\lambda_m\xi)$, quantity defined by equation (6).

Greek symbols
ρ, fluid density;
μ, coefficient of shear viscosity;
ν, kinematic viscosity (μ/ρ);
λ, separation constant;
$\bar{\alpha}_1, \bar{\alpha}_2, \bar{\alpha}_3$, variables defined by equations (11)–(13) respectively;
Γ_1, Γ_2, constants defined by equations (9) and (10).

Subscripts
θ, angular component;
s, steady state solution;
v, solution in the presence of viscous dissipation of heat;
w, condition at the walls;
∞, solution at time $t = \infty$.

INTRODUCTION

OVER the past few years, transpiration cooling has been theoretically analyzed by several investigators in various geometries [1–3]. The theoretical knowledge of this method of cooling is of significant importance in space industry, combustion chamber walls, exhaust nozzles and porous walled reactors. The purpose of this note is to present analytical solutions for temperature distributions in steady as well as unsteady state heat transfer in laminar tangential flow between two infinite, coaxial cylinders in the presence of radial flow (see Fig. 1). The solutions are obtained in the absence as well as presence of viscous dissipation of heat. The results should be useful in evaluating the effectiveness of transpiration cooling of a wide variety of fluids in the Couette flow system shown in Fig. 1.

THEORETICAL

For the system illustrated by Fig. 1, at time $t < 0$, both